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Technical Note

On the effect of lateral thermal convection on freely falling liquid film flow

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1. Introduction

Because of the importance of falling liquid films in engineering applications such as evaporators, chemical reactors, condensers, turbine blades, etc., film flow and related problems have received considerable attention. Great augmentation in heat transfer can be expected for a vertical wall being wetted by a liquid film. So, a large body of literature was presented or published on the topic of transport characteristics of falling liquid film flow, especially in the last two decades. Chu and Dukler [1, 2] measured instantaneous thickness related to wavy liquid film flow in the situation of $Re > 700$. They stressed the random characteristics of film thickness variation with time and presented meaningful statistical analyses. They interpreted their data in terms of two main classes of random waves, i.e., large waves which carry a significant portion of all flowing liquid and small waves which cover the substrate that exists between large waves. They concluded that, the small wave structure controls the fluid resistance and transfer process in the gas while the large waves control these same processes in the liquid film. Recently, instantaneous film thickness and other parameters were studied experimentally for Re ranging from 30 to 5000 [3–5], or studied numerically [6–9]. Several conclusions were reached as follows:

(1) The film thickness of wavy liquid film is instantaneous and irregular, so does the velocity and temperature within the film;
(2) There are circulation regions under large waves;
(3) The wave patterns are very complicated and random.

The effort on this topic in past several decades affords us the opportunity to realize some fundamental aspects of the freely falling liquid film flow phenomena. Unfortunately, there still exists serious gaps in our knowledge which hamper us to describe such complicated phenomena properly. The main deficiency might be that a proper dimensionless parameter is not employed to describe the process and physical nature. Until now, Reynolds number is used as a dominative parameter by researchers. However, there will not be a unified criterion about wavy and turbulent transition points using this parameter study. For example, several researchers [6, 10, 11] reported the onset of wavy motion occurred in the situation for Reynolds number less than 10. Rohsenow et al. [12] and Mudawar [13] suggested that, laminar for $Re < 33$, wavy laminar for $33 < Re < 1600$, and turbulent for $Re > 1600$. Nagasaki and Hijikata [14], Wasden and Dukler [15] declared that there is a laminar circulation occurs for Reynolds numbers $Re = 30$, which means the film is in wavy state as $Re \sim 30$. Karapantsios et al. [16] claimed that the film flow at $Re = 509$ is usually considered to be laminar. These show that the film waves onset at significantly different Reynolds number. Another important deficiency lies on that, in the previous researches, the thermal conditions of vertical walls or vertical cylinders were not clearly specified, and hence, a question comes if the thermal boundary conditions effect on the momentum transport process. Besides, how can we define exactly the so-called laminar, wavy and turbulent film flow. We have a clear description for gas flow along plates about the behavior of boundary layer flow and turbulent layer flow. But for wavy and turbulent film flow, such a clear description is not found in literature.

Heretofore there is no unified theoretical description of the thin freely falling liquid film flow. Statistical theory was used to describe the distribution of instantaneous thickness and others, and the probability density function was based on the experimental measurement of film

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thickness. However, the transport mechanism is shaded by the statistical theory, for it is impossible to insight into the intrinsic reason for the momentum and heat transport within liquid film using the theory. It is questionable whether there are unified probability density function distributions from different experiments reported in literature for wide range of *Re*.

Apparently, theoretical and experimental investigations are still needed to understand the transport phenomenon of thin liquid film flow. Here, a proper criterion parameter will be proposed by analyzing the effect of lateral thermal convection on momentum and heat transport process and develop the simplified momentum and energy equations.

2. Analysis

Liquid film flow on a vertical wall is driven by gravity. Meanwhile, there are thermal inequalities within the film because of the temperature difference between the wall and the free interface. For this respect, Rayleigh number, *Ra*, could be introduced based on the general thermal condition of the thin film flow. The calculated values of *Ra* for falling water film, with assumed film thickness $\delta = 1$ mm or $\delta = 2$ mm and assumed temperature difference between the vertical wall and free interface varying from 10–100°C are summarized in Table 1. Comparing with the corresponding Reynolds number, ranging from about ten to thousand, the variation of *Ra* is much greater than that of the onset *Re* mentioned by the above mentioned researches. The lateral thermal convective function is prominent in freely falling liquid film flow along vertical walls, so that the lateral thermal convection may result in significant influence or impact on the liquid film flow along the wall if there exists a larger temperature difference between the vertical wall and free interface. In the situation of practical application with a large amount of heat flux conducted through the wall and a great amount of latent heat issued by evaporation on the free interface, the lateral thermal convection may play the critical role in the momentum and heat transport within the liquid film.

Recently, papers were presented for investigating the transport phenomena in the thin freely falling liquid film flow and instability analysis by numerical simulation [7, 8]. The buoyancy force was taken into consideration, but the governing equations were reduced on the base of Reynolds criterion so that the momentum and energy equations were not conjugated and the effect of the lateral thermal convection was ignored.

Consider a two-dimensional freely falling liquid film flow along a vertical wall and with a free interface. The continue, momentum and energy equations are given as:

$$\text{Continuity:}\quad \frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\text{Momentum:}\quad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\nu\nabla^2 u \tag{2}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\boldsymbol{g}\beta T+\nu\nabla^2 w \tag{3}$$

$$\text{Energy:}\quad \frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}=\alpha\nabla^2 T. \tag{4}$$

Similar to [19], the conjugated stream function and temperature equations can be reduced as follows:

$$\nabla^{*2}\frac{\partial\psi^*}{\partial t^*}+\frac{\partial(\psi^*,\nabla^*\psi^*)}{\partial(x^*,z^*)}-Pr\frac{\partial\theta^*}{\partial x^*}+\frac{\boldsymbol{g}\beta\delta^3\Delta T}{\alpha^2}-Pr\,\nabla^{*4}\psi^*=0 \tag{5}$$

$$\frac{\partial\theta^*}{\partial t^*}+\frac{\partial(\psi^*,\theta^*)}{\partial(x^*,z^*)}+Ra\frac{\partial\psi^*}{\partial z^*}-\nabla^{*2}\theta^*=0 \tag{6}$$

with Jacobian operator:

$$\frac{\partial(\psi^*,\theta^*)}{\partial(x^*,z^*)}=\left(\frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial z^*}-\frac{\partial\theta^*}{\partial x^*}\frac{\partial\psi^*}{\partial z^*}\right). \tag{7}$$

Here, the superscript "*" presents dimensionless values, ψ is stream function, θ is temperature departure, α is the thermal diffusivity, β is volumetric expansion coefficient, $\boldsymbol{g}$ is gravitational acceleration, ΔT is the temperature

Table 1
The Rayleigh numbers of freely falling liquid films (for water)

	ΔT (°C)							
	20	30	40	50	60	70	80	90
ν ($m^2\,s^{-1}$) $\times 10^6$	1.006	0.919	0.832	0.745	0.658	0.613	0.568	0.523
α ($m^2\,s^{-1}$) $\times 10^7$	1.430	1.450	1.471	1.492	1.512	1.523	1.533	1.543
Ra, for $\delta = 1$ mm	245.2	397.1	576.5	793.5	1063.8	1322.6	1620.7	1966.7
$\delta = 2$ mm	1961.9	3177.1	4612.2	6347.9	8510.9	10 581.0	12 965.5	15 733.0

Where $Ra = \boldsymbol{g}\beta\Delta T\delta^3/\nu\alpha$, the temperature at the free interface is assumed as $T_i = 10$°C and the average temperature $\bar{T}$ within the films is $\bar{T} = T_i+\Delta T/2$, or $T_w = (T_i+\Delta T)$ is the temperature of the wall.

difference between the vertical wall and free interface, t represent time, δ is film thickness, Pr is the Prandtl number, Ra is the Rayleigh number.

3. Conclusions

The above mentioned governing equations for liquid film flow are conjugated, so that the momentum and thermal convection are correlated. The critical parameters are Prandtl number, Pr, and Rayleigh number, Ra, rather than Re which is conventionally adapted in literature. In a wide range of liquid film flow conditions, Pr can be considered as a constant value for a given liquid so that Ra will be the only critical parameter for a specified liquid. This implies that for the thin freely falling liquid film flow, the lateral thermal convection between the wall and free boundary could affect the film layer stability.

Rayleigh number, Ra, might be taken as a criteria for the film flow changing from smooth to wavy or from wavy to chaos. This should be demonstrated further by theoretical and experimental investigations.

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